# %Chapter Fundmental Knowledge

%introduction

Before the discussion about coupling fibers to waveguides it is necessary to present some basic knowledge involved in this work. In this chapter lens theory and fiber optics will be as common knowledge at first introduced. Then the description of Gaussian beam is helpful for understanding some terms, such as spot size, in the following research.

In order to analyze the simulations results from CST MWS, we also have to present finite integral method (FIT), which is implemented in CTS MWS. At last in this work S-Parameters will be described in this chapter because the $|S\_{21}|$ is generally used to measure the coupling efficiency.

## %\section{Optics}

### \subsectioin{Lens Theory}

%Lens optics

Lens is widely applied in optics. Some are used for photography of camera; some are used for microscopes and telescopes. In this work lens is used for coupling fibers to photonic waveguide. No matter what kind of application they all base on primary rules: lens theory.\\

One important function of lens is focus. It is a common experience by using a magnifying glass to set small pieces of paper or leaves on fire with sunlight. In this scene the lens of the magnifying glass focuses all incoming sun rays at a single spot. This spot is called focal point and the distance between the focal point and the center of the lens is called focal length $f$. The dimensions of this spot and the focal length are two primary characters of the lens and values of them depend on the radium of curvature of the lens surface and on the refractive index of the material the lens is made from. \\

\begin{figure}[httbp]

\centering

\includegraphics[width=0.6\textwidth]{bilder/lens\_define}

\caption{The quantities define of singlet lenses \cite{lens\_theory\_LC\_Ltd}}

\label{fig:lens\_define}

\end{figure}

In order to understand more specifics about lenses Fig.\ref{fig:lens\_define} presents a singlet lens with geometric parameters. \”The optical axis (O-O')of the lens is a line passing through the centers of curvature of the two spherical lens surfaces. \”

Rays A and B, parallel with the optic axis (O-O'), are casted from different two sides through the lens and respectively cross the axis at their front and back focal points $F1$ and $F2$. The front and back principal points $H1$ and $H2$ are the intersections of the optical axis with the front and back principal surfaces. Points $V1$ and $V2$ are called the front and back vertices respectively\cite{lens\_theory\_LC\_Ltd}. Some important quantities are defined in Tab. \ref{tab:lens\_quantities}. Here the distance $t\_{c}$ is the center thickness of the lens $V1V2$.\\

\begin{table}

\centering

\caption{Important Quantities for Singlet Lenses Immersed in Air \cite{lens\_theory\_LC\_Ltd}}

\begin{tabular}{|c|c|c|}

\hline

\textbf{Symbol}&\textbf{Description}&\textbf{Formular}\\

\hline

$f$ & \parbox[c]{6cm}{

\begin{center}

effective focal length

\end{center}

}& $\frac{1}{f}=(n-1)\left[\frac{1}{R\_{1}}-\frac{1}{R\_{2}} \right]+\frac{t\_{c}(n-1)^2}{nR\_{1}R\_{2}}$ \\

\hline

$BFD$ &\parbox[c]{6cm}{

\begin{center}

back focal distance

\end{center}

}& $BFD=f\left[ 1-\frac{t\_{c}(n-1)}{nR\_{1}}\right]$ \\

\hline

$FFD$ &\parbox[c]{6cm}{

\begin{center}

front focal distance

\end{center}

}& $FFD=f\left[ 1+\frac{t\_{c}(n-1)}{nR\_{1}}\right]$ \\

\hline

$H2V2$ & \parbox[c]{6cm}{

\begin{center}

back vertex to back principal point distance

\end{center}

} & $H\_{2}V\_{2}=f-BFD=-f\frac{t\_{c}(n-1)}{nR\_{1}}$ \\

\hline

$V1H1$ & \parbox[c]{6cm}{

\begin{center}

front vertex to front principal point distance

\end{center}

} & $V\_{1}H\_{1}=f-FFD=-f\frac{t\_{c}(n-1)}{nR\_{2}}$ \\

\hline

\end{tabular}

\label{tab:lens\_quantities}

\end{table}

The classical singlet lenses are plano convex, plano concave, equiconvex and equiconcave lenses, which are listed in following Tab.\ref{tab:lenses\_focal\_length} with their focal length relation.\\

\begin{table}

\centering

\begin{tabular}{|c|c|c|}

\hline

\textbf{Type}&\textbf{Description}&\textbf{Formular}\\

\hline

Plano Convex & \parbox[c]{2.1cm}{\includegraphics[width=2cm]{bilder/plano\_convex}}& $f=\frac{R}{(n-1)}$ \\

\hline

Plano Concave &\parbox[c]{2.1cm}{\includegraphics[width=2cm]{bilder/plano\_concave}} & $f=-\frac{R}{(n-1)}$ \\

\hline

Equiconvex & \parbox[c]{2.1cm}{\includegraphics[width=2cm]{bilder/equi\_convex}} & $f=\left[\frac{2(n-1)}{R} - \frac{t\_{c}(n-1)^2}{nR^2}\right]^{-1}$ \\

\hline

Equiconcave & \parbox[c]{2.1cm}{\includegraphics[width=2cm]{bilder/equi\_concave}} & $f=\left[\frac{2(n-1)}{R} + \frac{t\_{c}(n-1)^2}{nR^2}\right]^{-1}$ \\

\hline

\end{tabular}

\caption{Focal length Formulas of Simple Singlet Lenses in Air.

The Radii are considered positive in the formulas below.}

\label{tab:lenses\_focal\_length}

\end{table}

The performance of a lens is usually estimated by spot size and focal lengths. The definition of the spot size will be declared later in section \ref{sect:gaussian\_beam}. The focal length is in common sense the distance from lens center to minimum spot. But this idea is not exact in any case. Here we can testify this thought by Fig.\ref {fig:focal\_length} and following equations (\ref{eq:snell\_focal}-\ref{eq:focal\_length}).\\

\begin{figure}[!ht]

\centering

\includegraphics[width=0.6\textwidth]{bilder/focal\_length}

\caption{Schema of refraction of parallel light by lens.}

\label{fig:focal\_length}

\end{figure}

\begin{equation}

nsin\theta=sin\psi

\label{eq:snell\_focal}

\end{equation}

\begin{equation}

\phi=\psi-\theta

\label{eq:psi\_phi}

\end{equation}

\begin{align}

L&=Rsin(\theta) ctan(\phi) \nonumber\\

&=Rsin(\theta)\farc{cos(\psi-\theta)}{ sin(\psi-\theta)} \nonumber\\

&= Rsin(\theta)\farc{cos(\psi)cos(\theta)+sin(\psi)sin(\theta)}{sin(\psi)cos(\theta)-cos(\psi)sin(\theta)} \nonumber\\

&= Rsin(\theta)\farc{cos(\psi)cos(\theta)+nsin^{2}(\theta)}{nsin(\theta)cos(\theta)-cos(\psi)sin(\theta)} \nonumber\\

&=R\farc{cos(\psi)cos(\theta)+nsin^{2}(\theta)}{ncos(\theta)-cos(\psi)} \nonumber\\

&=R\farc{cos(\psi)cos(\theta)-ncos^{2}(\theta)+n}{ncos(\theta)-cos(\psi)} \nonumber\\

&=R \left[-cos(\theta)+\frac{n}{ncos(\theta)-\sqrt{1-n^{2}sin^{2}(\theta)}} \right]

\label{eq:focal\_length}

\end{align}

In Fig. \ref{fig:local\_length} there is a lens with radium $R$ and index $n$. O-O’ axis goes through the lens center. The light source $a$ emits a ray parallel with the O-O’ axis and at the point $b$ on the lens surface is refracted. At last the refracted ray cross the O-O’ axis at point F. $\theta$ is the input angle, \psi the output angle and \phi the angle from output ray to O-O’ axis. According the SNELL’s LAW we get the relation between \theta and \psi in (ref{eq:snell\_focal}). \phi and \psi has the relation like (\ref{eq:psi\_phi}). Then the distance $L$ from point H to F is given by (\ref{eq:focal\_length}). When $\theta$ is close to 0 like (\ref{ eq:focal\_length\_app}) $L$ is right equal the formula of Plano Convex in Tab. \ref{tab:lenses\_focal\_length }. So this formula of focal length is only valid for a small angle lens.

\begin{equation}

L=R\left[ -1+\frac{n}{n-1}\right]=\frac{R}{n-1}

\label{eq:focal\_length\_app}

\end{equation}

Where is the focal point indeed located? \cite{lens\_theory\_LC\_Ltd} has refered that the minimum spot lies between the maeginal plane and paraxial focal plane. All the distances which are in following discussed base on the assumption that the back vertex (V2) of the lens is regarded as origin. In Fig.\ref{fig:min\_max\_spot} there are "'geometrical traces of 25 rays in the focal region 100mm focal length plano convex lens(n=1.515)."' "'The rays are launched parallel to the axis (O-O') and equally spaced in a region above and below the axis in a plane containing the axis(\textbf{meridional plane})"'. The marginal plane (\textbf{MP}) goes through the focal point of marginal rays. The paraxial focal plane (\textbf{PP}) goes through the focal point of paraxial rays. "'The distance from \textbf{PP} to \textbf{MP} is the \textbf{longitudinal aberration LAm}"'. The minimum spot (\textbf{MS}) is located at the plane, which is approximately 3/4 LAm back toward the lens from the \textbf{PP}.

\begin{figure}[httbp]

\centering

\includegraphics[width=0.6\textwidth]{bilder/min\_max\_spot}

\caption{Schema to estimating the minimum spot location \cite{lens\_theory\_LC\_Ltd}.}

\label{fig:min\_max\_spot}

\end{figure}

### \subsectioin{Optical waveguides and Fibers}

%Optical waveguides

For the transmission of optical signal optical waveguides are applied. The general waveguides are semiconductor waveguide and optical fibers.

Fig. \ref{fig:semi\_waveguides} shows two semiconductor waveguides commonly used in integrated optics. Rib waveguide is composed of a rib guide on a substrate ($n=n\_{2}$). Buried waveguide is a high index guide ($n=n\_{1}$) surrounded by low index cladding ($n=n\_{2}$).\\

\begin{figure}

\centering

\subfigure[Rib waveguide]{

\includegraphics[width=0.4\textwidth]{bilder/approxmate\_waveguide}

\label{fig:semi\_rib\_waveguide}

}

\hfill

\subfigure[Buried waveguide]{

\includegraphics[width=0.4\textwidth]{bilder/buried\_waveguide}

\label{fig:semi\_buried\_waveguide}

}

\caption{Schema of semiconductor waveguides}

\label{fig:semi\_waveguides}

\end{figure}

Optical fibers are widely used for telecommunication and data networks. The Fig.\ref{fig:opticfiber} presents a simplest optical fiber and how lights propagate in the fiber . Optical fiber typically consists of a transparent core with index $n\_{1}$ surrounded by a transparent cladding material with a lower index of refraction $n\_{2}$.

\begin{figure}[httbp]

\centering

\includegraphics[width=0.8\textwidth]{bilder/opticfiber}

\caption{linght refraction in optic fibers}

\label{fig:opticfiber}

\end{figure}

\textbf{ Total Reflection}\\

\begin{figure}[!ht]

\centering

\subfigure[totalreflection]{

\includegraphics[width=0.3\textwidth]{bilder/totalreflection01}

\label{fig:totalreflection01}

}

\hfill

\subfigure[Buried waveguide]{

\includegraphics[width=0.3\textwidth]{bilder/totalreflection02}

\label{fig:totalreflection02}

}

\hfill

\subfigure[Buried waveguide]{

\includegraphics[width=0.3\textwidth]{bilder/totalreflection03}

\label{fig:totalreflection03}

}

\caption{Total reflection}

\label{fig:totalreflection}

\end{figure}

Whatever semiconductor waveguides or optical fibers, the principle of the light propagation in waveguides is total reflection. The principle of the total reflection is explained in \cite{optical\_waveguides\_fibers} with Snell's law. In Fig.\ref{fig:totalreflection01} the input light strikes the boundary between two different isotropic media with respective refractive index $n\_{1}$ and $n\_{2}$. Where $theta\_{1}$ is incidence angle, $\theta\_{2}$ refractive angle and $\theta\_{r}$ reflective angle. Through SNELL's Law there are relations (\ref{eq:snell}-\ref{eq:reflection}). For $n\_{1}<n\_{2}$ there is always a relation $\theta\_{1}>\theta\_{2}$. If the refractive indexes has the relation $n\_{1}>n\_{2}$, then the incidence angle $\theta\_{1}$ is narrower than the refractive angle $\theta\_{2}$ like Fig. \ref{fig:totalreflection02}. If the incidence angle is increased wider than a critical angle $\theta\_{c}$ (\ref{eq:critical\_angle}) there will be no light passing through the boundary and all of the lights are reflected like Fig. \ref{fig:totalreflection03}. This phenomenon is so called total reflection.

\begin{align}

n\_{1}sin\theta\_{1}&=n\_{2}sin\theta\_{2}

\label{eq:snell}\\

\theta\_{1}=\theta{r}

\label{eq:reflection}

\end{align}

\begin{equation}

\theta\_{c}=arcsin(\frac{n\_{2}}{n\_{1}})

\label{eq:critical\_angle}

\end{equation}

%%dispersion

\textbf{ Numerical Apertur }\\

%Numerical Aperture

Another important character of optical waveguides is numerical aperture. Back to the Fig.\ref{fig:opticfiber} the incidence beam originate from the air into the fiber. There is a maximum coupling angle, so that the beam can be guided under the total reflecting conditions. Its sinus value (\ref{eq:NA}) is called \textbf{Numerical Apertur(NA)}, which indicate the acceptable range of ray beams.

\begin{align}

sin\theta\_{i}&=\frac{n\_{1}}{n\_{0}}sin(90^{o}-\theta\_{c})=n\_{1}cos\theta\_{c} \nonumber\\

&=n\_{1}\sqrt{1-sin^{2}\theta\_{c}}=n\_{1}\sqrt{1-\left(\frac{n\_{2}}{n\_{1}}\right)^2}=\sqrt{n^2\_{1}-n^2\_{2}}

\label{eq:NA}

\end{align}

\textbf{Mode of the waveguide}\\

"' An eigenmode $m$ of a waveguide structure is a propagation or evanescent wave which maintains its transverse shape during propagation "'\cite{integrated\_optics}. The eigenmode of a waveguide can be presented as (\ref{eq:e\_eigenmode}-\ref{eq:h\_eigenmode}).

\begin{align}

E^{m}(r\_{t},z)&=E^{m}\_{0}(r\_{t})e^{jk\_{m}z}

\label{eq:e\_eigenmode}\\

H^{m}(r\_{t},z)&=H^{m}\_{0}(r\_{t})e^{jk\_{m}z}

\label{eq:h\_eigenmode}

\end{align}

Where $k\_{m}$ is the propagation constant. When the request $k\_{0}n\_{1}>k\_{m}>k\_{0}n\_{2}$ is matched this mode is a guided mode for the waveguides \cite{script\_FT\_TET}. In this work the coupling bases on the fundamental mode.

## \Section{Gaussian Beam}

In the world there is no natural source of paraxial ray. Each beam of lights can be considered originating from a simple origin: point light source, which emits anisotropic light in all directions. Thus a normal light source cannot provide perfect focused beams for optical applications.

TEM$\_{00}$ mode of a laser source is a perfect plane wave with Gaussian transverse irradiance profile\cite{CVI\_Melles\_Griot\_Technical\_Guide}. Therefore the laser light is considered as a beam propagating in a well-defined direction with limited spreading in transversal dimensions.\\

In \cite{ script\_FT\_TET} the characteristics of Gaussian beams are described. The transversal components of the field is presented as (\ref{eq:gaussian\_01}).

\begin{equation}

E\_{x}=\psi(x,y,z)e^{-jk\_{0}nz}

\label{eq:gaussian\_01}

\end{equation}.

Partial differential form is given by (\ref{eq:gaussian\_02}). Because the paraxial beams spread slowly in transversal direction due to the z-axis, the term $\frac{\partial ^{2}\psi}{\partial z^2}$ is very smaller than other terms. Thus equation (\ref{ eq:gaussian\_02}) become an approximation (\ref{ eq:gaussian\_03}).

\begin{eqaution}

\Delta E\_{x}=\frac{ \partial ^{2}\psi}{\partial x^2}+\frac{ \partial ^{2}\psi}{\partial y^2}+\frac{ \partial ^{2}\psi}{\partial z^2}-2jk\_{0}n\frac{\partial\psi}{\partial z}=0

\label{eq:gaussian\_02}

\end{equation}

\begin{eqaution}

\frac{ \partial ^{2}\psi}{\partial x^2}+\frac{ \partial ^{2}\psi}{\partial y^2}-2jk\_{0}n\frac{\partial\psi}{\partial z}=0

\label{eq:gaussian\_03}

\end{equation}

The equation(\ref{eq:gaussian\_03}) can also be transformed into cylinder coordinate and become (\ref{ eq:gaussian\_04})

\begin{eqaution}

\frac{1}{r}\frac{ \partial }{\partial r}\left(r\frac{\partial z}{\partial r}\right)+\frac{1}{r^2}\frac{ \partial ^{2}\psi}{\partial \phi^2}-2jk\_{0}n\frac{\partial\psi}{\partial z}=0

\label{eq:gaussian\_04}

\end{equation}

(\ref{eq:gaussian\_05}) is one general solution of (\ref{eq:gaussian\_04})

\begin{eqaution}

\psi{r,z}=\psi\_{0}exp\left(-j\left[P(z)+2 \frac{ k\_{0}n}{2q(z)}r^2\right]\right)

\label{eq:gaussian\_05}

\end{equation}

Where $P(z)$ and $q(z)$ meet the equation (\ref{eq:gaussian\_04}) for any $r$. The term $q(z)$ has also the realation(\ref{}) with some physical variables $w(z)$ and $R(z)$

\begin{equation}

\frac{1}{q(z)}=\frac{1}{R(z)}-\frac{j\lambda}{\pi nw^{2}(z)}

\label{eq:gaussian\_06}

\end{equation}

Where $w(z)$ is the 1/e^2 irradiance radius due to propagating distance z and $R(z)$ is the wavefront radius of curvature due to propagating distance z

\begin{equation}

W(z)=w\_{0}\sqrt{1+(\frac{\lambda z}{n\pi w^{2}\_{0}})^{2}}

\label{eq:gaussian \_07}

\end{equation}

\begin{equation}

R(z)=z\left[1+(\frac{\lambda z}{n\pi w^{2}\_{0}})^{2}\right]

\label{eq:gaussian\_08 }

\end{equation}

If (\ref{gaussian\_06}) is inserted in (\ref{ gaussian\_05}) then we can get a Gauss form result like (\ref{ gaussian\_09}). That means the field of a Gaussian beam is in transversal dimensions a Gaussian distribution like Fig. \ref{fig:Gaussian\_verteilung}.

\begin{equation}

\psi(r,z)=w\_{0}exp\left(-\frac{r^2}{w^2\_{0}}\right)

\label{eq:gaussian\_09}

\end{equation}

\begin{figure}[!ht]

\includegraphics[width=0.8\textwidth]{bilder/gussian\_verteilung}

\caption{Transversal profie of the Gussian beam amplitude at the beam waist (dashed line) and irradiance (solid line). Both of them have been normalized to the maximum value. The value of the width of the beam waist $\omega\_{0}$ is 0.1 mm. The horizontal lines represent (in increasing value)the $1/e^{2}$ of the maximum irradiance, the $1/e$ of the maximum amplitude, and the 0.5 of the maximum irrance and amplitude.}

\label{fig:gussian\_verteilung}

\end{figure}

According to definitions of $w(z)$ and $R(z) in ($\ref{eq:gaussian\_07}-\ref{eq:gaussian\_08}) the longitudinal profile of Gaussian beams can be drawn as Fig. \ref{ gussian\_profile}.

\begin{figure}[!ht]

\includegraphics[width=0.8\textwidth]{bilder/gussian\_profile}

\label{fig: gussian\_profile}

\end{figure}

Where the beam radius $w(z)$ is the distance from the beam axis where the intensity drops to $1/e2 (\sim13.5\%)$ of the maximum value. A hard aperture with radius w can transmit $\sim86.5\%$ of the optical power. For an aperture radius of $1.5 w$ or $2 w$, this fraction is increased to $98.9\%$ and $99.97\%$, respectively.

%Spot Size

\textbf{Spot Size}\\

Another important characteristic of Gaussian beams is \textbf{Spot Size}.In a cross-section of a gaussian beam the beam intesity is approximately distibuted as gaussian function. The spot size is the diameter of a area at whose edge the value of the electrical field intensity decay to $1/e$ of its peak value, otherwise the energy density to $1/e^2$ of the peak value.